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E-mail :
editor.ijpast@gmail.com
editor@ijpast.in

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Regarding the Connection Between Casimir Forces and Statistical Correlations

O.Vamsi¹, Ch.Manohar²,

Abstract

Using a microscopic viewpoint, we demonstrate that the Casimir force can be conveniently stated in terms of the bulk one-particle density matrix for a perfect quantum gas contained in a slit. Casimir force range may be related to the bulk correlation length using the appropriate formula, which is valid for both bosons and fermions. The Casimir forces' low-temperature behaviour is derived.

Introduction

Many investigations [1–17] have focused on the Casimir forces operating on perfectly horizontal walls filled with a perfect quantum gas (the slit geometry). There was still no solution to the outstanding subject of why the range of Casimir forces and the bulk correlation length behave so similarly at the microscopic level. It is our intent to shed light on this interesting and physiologically significant topic. Even though the Casimir forces are boundary-dependent, we shall demonstrate that in the thermodynamic limit they may be described

easily in terms of the one-particle density matrix [1,18].

It itself is independent of the boundedness constraints. The distance between two locations in space, denoted by $r_{12} = |r_1 - r_2|$. Density, or number, of particles in a gas that is otherwise uniform is given by $\rho = 1/\lambda^3$. Diagonal components of the two-particle reduced density matrix [1,6,18] provide the two-particle number density $\rho_2(r_{12})$ of pairs of particles separated by distance r_{12} .

Assistant Professor ^{1,2},
Mail Id : oodavamsi888@gmail.com,
Department of Physics,
PBR VISVODAYA INSTITUTE OF TECHNOLOGY AND SCIENCE, KAVALI.

$$\langle \Gamma_1, \Gamma_2 | \hat{\rho}_2 | \Gamma_1, \Gamma_2 \rangle \equiv \rho_2(r_{12}) \quad (2)$$

The pair correlation function is defined as the ratio of the square root of two to one.

$$\chi_2(r) = \rho_2(r) - \rho^2 \quad (3)$$

Correlation function $B_2(r)$ for a perfect Bose gas may be written entirely in terms of $B_1(r)$ [1].

$$\chi_2^B(r) = \begin{cases} [\rho_1^B(r)]^2 & \text{if } \rho < \rho_c \\ [\rho_1^B(r)]^2 - (\rho - \rho_c)^2 & \text{if } \rho \geq \rho_c \end{cases} \quad (4)$$

indicating the critical density for Bose-Einstein condensation, c . Take note that the shape of the function $B_1(r)$ varies depending on the value of c and $>c$. For a perfect Fermi gas, an even more elementary relationship holds. [18]

$$\chi_2^F(r) = -[\rho_1^F(r)]^2 \quad (5)$$

In the next sections, we will explore the explicit formulations for $B_1(r)$ and $F_1(r)$. It is shown in Section 2 how to derive an accurate analytic connection between the Casimir forces and the one-particle function $1(r)$ that describes an infinite system. The deduced formula will serve as the foundation for further investigation. And perhaps most crucially, it allows us to determine a direct connection between the Casimir forces and the pair correlation function (r) . In fact, Eqs. (4) and (5) suggest a direct relationship between the Casimir forces and the correlation function since they can be written in terms of the one-particle density matrix (when writing these equations we do not display their spin dependence). So, from a microscopic perspective, the parallelism in the behaviour of the range of the forces and the bulk correlation length is readily explicable (Sect. 3). Our research makes it possible to recapture in a nutshell a wide range of narrow outcomes obtained elsewhere using other methods.

2 Relation Between Casimir Forces and the One-Particle Density Matrix

We think of the gas as consisting of identical particles with mass m , and we put them in a cube with sides L and length D , with a volume of $V = L^2D$. We settle on a reference frame with axes that run perpendicular to the boundaries of the box,

so that x , y , and z all point in the same direction. Specifically, the z axis is aligned in a direction that is perpendicular to the four sides of the square. For a given temperature T and chemical potential, the grand canonical free energy of bosons is given by the series $B(T, L, D, \mu)$.

$$\Omega^B(T, L, D, \mu) = k_B T \sum_{\mathbf{k}} \ln \left[1 - z \exp \left(-\frac{\lambda^2}{4\pi} k^2 \right) \right] \quad (6)$$

whereas for fermions

$$\Omega^F(T, L, D, \mu) = -k_B T \sum_{\mathbf{k}} \ln \left[1 + z \exp \left(-\frac{\lambda^2}{4\pi} k^2 \right) \right] \quad (7)$$

Thus, $\omega = h / \ln 1 - z \exp 2 k_4 k_2 \ln 1 + z \exp 2 k_4$. (6) (7) (7) Where $z = \exp(\mu/k_B T)$, k_B is the Boltzmann constant, and $\lambda = h / \sqrt{2m k_B T}$ is the de Broglie thermal wavelength. Allowed wave vectors \mathbf{k} are taken into account in the above summing, which extends to the whole domain defined by the boundary conditions. The spin of the particles is ignored for the sake of notational simplicity, as was noted in the introduction. On the x and y axes, we implement periodic boundary conditions, setting $0 \leq x, y < 2\pi n$ (where n is a positive integer between 0 and 2). Obtaining the limit using the division of Eq. (6) by L^2 The formula for the total grand canonical free energy density per unit wall area is obtained by considering indefinitely long square walls.

$$\omega^B(T, D, \mu) = \lim_{L \rightarrow \infty} \frac{\Omega^B(T, L, D, \mu)}{L^2} = k_B T \sum_{k_z} \iint \frac{dk_x dk_y}{(2\pi)^2} \ln \left[1 - z \exp \left[-\frac{\lambda^2}{4\pi} (k_x^2 + k_y^2 + k_z^2) \right] \right] \quad (8)$$

The corresponding formula for fermions reads

$$\omega^F(T, D, \mu) = -k_B T \sum_{k_z} \iint \frac{dk_x dk_y}{(2\pi)^2} \ln \left[1 + z \exp \left[-\frac{\lambda^2}{4\pi} (k_x^2 + k_y^2 + k_z^2) \right] \right] \quad (9)$$

The formula for the total grand canonical free energy density per unit wall area is obtained by multiplying Eq. (6) by L^2 , and then taking the limit L of indefinitely extended square walls.

$$\begin{aligned} \omega^B(T, D, \mu) &= \lim_{L \rightarrow \infty} \frac{\Omega^B(T, L, D, \mu)}{L^2} \\ &= k_B T \sum_{k_x} \iint \frac{dk_y dk_z}{(2\pi)^2} \ln \left[1 - z \exp \left[-\frac{\lambda^2}{4\pi} (k_x^2 + k_y^2 + k_z^2) \right] \right]. \end{aligned} \quad (8)$$

The corresponding formula for fermions reads

$$\omega^F(T, D, \mu) = -k_B T \sum_{k_x} \iint \frac{dk_y dk_z}{(2\pi)^2} \ln \left[1 + z \exp \left[-\frac{\lambda^2}{4\pi} (k_x^2 + k_y^2 + k_z^2) \right] \right]. \quad (9)$$

The Dirichlet boundary conditions are appropriate for hard walls. However, for complex systems, we will consider three types of boundary conditions in the direction:

$$\omega_{per}^B(T, 2D, \mu) = \sigma^B(T, \mu) + 2\omega_{Dir}^B(T, D, \mu) = -\sigma^B(T, \mu) + 2\omega_{Neu}^B(T, D, \mu) \quad (10)$$

where

$$\sigma^B(T, \mu) = k_B T \iint \frac{dk_x dk_y}{(2\pi)^2} \ln \left[1 - z \exp \left[-\frac{\lambda^2}{4\pi} (k_x^2 + k_y^2) \right] \right] = -\frac{k_B T}{\lambda^2} g_2(e^{\mu/k_B T}) \quad (11)$$

Coarse and smooth contribute in form $\omega = 0$ in the series appearing in Eq. (8). In the conventional definition, the Bose function $g_2(z)$ is written as follows: $g_2(z) = \sum_{l=1}^{\infty} z^l / l^2$. We remark that the surface tension coefficient of an ideal Bose gas, denoted by B Neu, Dir(T, μ), corresponds to the Neumann or Dirichlet boundary conditions, respectively [13]. Similarly, for fermion we find.

$$\omega_{per}^F(T, 2D, \mu) = \sigma^F(T, \mu) + 2\omega_{Dir}^F(T, D, \mu) = -\sigma^F(T, \mu) + 2\omega_{Neu}^F(T, D, \mu), \quad (12)$$

where

$$\sigma^F(T, \mu) = -k_B T \iint \frac{dk_x dk_y}{(2\pi)^2} \ln \left[1 + z \exp \left[-\frac{\lambda^2}{4\pi} (k_x^2 + k_y^2) \right] \right] = \frac{k_B T}{\lambda^2} g_2(-e^{\mu/k_B T}). \quad (13)$$

Similar to the bosonic situation, we determine formulas for the anisotropic surface tension coefficients: If F Dir(T, μ) = F Neu(T, μ) = $-k_B T g_2(-a/k_B T) / 4\lambda^2$. Our attention now shifts to the derivation of the formula for the Casimir force $F(T, D, \mu)$ (or Casimir pressure) in terms of the one-particle density matrix. By definition

$$F(T, D, \mu) = -\frac{\partial}{\partial D} \omega_s(T, D, \mu), \quad (14)$$

where $\omega_s(T, D, \mu)$ is the screened energy density

$$\omega_s(T, D, \mu) = \omega(T, D, \mu) - D\omega_b(T, \mu) \quad (15)$$

Eq. (8) evaluates the density $B(T, D, \mu)$ under periodic boundary conditions and finds it to be equal to the difference between the total grand canonical free energy density per unit wall area and the bulk free energy density $b(T, D, \mu)$ (evaluated in the thermodynamic limit).

$$\begin{aligned} \omega_{per}^B(T, D, \mu) &= k_B T \sum_{n=-\infty}^{+\infty} \iint \frac{dk_x dk_y}{(2\pi)^2} \ln \left[1 - z \exp \left(-\frac{\lambda^2}{4\pi} \left[k_x^2 + k_y^2 + \left(\frac{2\pi n}{D} \right)^2 \right] \right) \right] \\ &= \sigma^B(T, \mu) + 2k_B T \sum_{n=1}^{+\infty} \iint \frac{dk_x dk_y}{(2\pi)^2} \ln \left[1 - z \exp \left(-\frac{\lambda^2}{4\pi} \left[k_x^2 + k_y^2 + \left(\frac{2\pi n}{D} \right)^2 \right] \right) \right]. \end{aligned} \quad (16)$$

At this point we will use the non-expanded form of the Euler-Maclaurin formula [14]

$$\sum_{n=1}^N f(n) = \int_0^N f(x) dx + \frac{1}{2} [f(N) + f(0)] + \int_0^N \left(x - [x] - \frac{1}{2} \right) f'(x) dx, \quad (17)$$

where $[x]$ denotes the largest integer not exceeding x , and $\{x\}$ is the fractional part of x . The period and properties of the function $(x - [x]) / 2$ shown in the integral and above are as follows: Fourier series representation.

$$x - [x] - \frac{1}{2} = -\frac{1}{\pi} \sum_{p=1}^{\infty} \frac{\sin(2p\pi x)}{p}. \quad (18)$$

Using Eqs. (17) and (18) we find

$$\begin{aligned} \omega_{per}^B(T, D, \mu) - D\omega_b(T, \mu) &= 2k_B T \iint \frac{dk_x dk_y}{(2\pi)^2} \int_0^{\infty} dx \left(x - [x] - \frac{1}{2} \right) \frac{\partial}{\partial x} \ln \left[1 - z \exp \left(-\frac{\lambda^2}{4\pi} \left[k_x^2 + k_y^2 + \left(\frac{2\pi x}{D} \right)^2 \right] \right) \right] \\ &= \lambda^2 k_B T \iiint \frac{dk}{(2\pi)^3} \left(\frac{Dk_x}{2\pi} - \left[\frac{Dk_x}{2\pi} \right] - \frac{1}{2} \right) \frac{k_x}{z^{-1} \exp(\lambda^2 k^2 / 4\pi) - 1} \\ &= -\frac{\lambda^2}{2\pi^2} \sum_{p=1}^{\infty} \iiint \frac{d\mathbf{k}}{(2\pi)^3} \frac{\sin(pDk_x)}{p} k_x \rho^B(\mathbf{k}) = \frac{\lambda^2}{2\pi^2} \sum_{p=1}^{\infty} \frac{\partial}{\partial D} \iiint \frac{d\mathbf{k}}{(2\pi)^3} \frac{\cos(pDk_x)}{p^2} \rho^B(\mathbf{k}) \end{aligned} \quad (19)$$

where $d\mathbf{k} = dk_x dk_y dk_z$, and $\rho^B(\mathbf{k}) \equiv \rho^B(k)$, $k = |\mathbf{k}|$, is the mean occupation number of the one-particle state \mathbf{k} .

The inverse Fourier transform of $\rho^B(\mathbf{k})$ is equal to the one-particle density matrix

$$\rho_1^B(r) = \iiint \frac{d\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} \rho^B(k) = \iiint \frac{d\mathbf{k}}{(2\pi)^3} \cos(\mathbf{k}\cdot\mathbf{r}) \rho^B(k). \quad (20)$$

We thus get from (19) the relation

$$\omega_{per}^B(T, D, \mu) - D\omega_D(T, \mu) = \frac{\hbar^2}{2\pi^2 m} \frac{\partial}{\partial D} \sum_{p=1}^{\infty} \frac{\rho_1^B(pD)}{p^2}. \quad (21)$$

Finally, using the defining Eq. (14) we arrive at the basic formula

$$F_{per}^B(T, D, \mu) = -\frac{\hbar^2}{2\pi^2 m} \frac{\partial^2}{\partial D^2} \sum_{p=1}^{\infty} \frac{\rho_1^B(pD)}{p^2} \quad (22)$$

Which relationship Casimir force and the one-particle density matrix is not to be expected to hold. For ideal Fermi gas, a similar computation yields the same relationship between the Casimir force and the one-particle density matrix F_1 . (pad)

$$F_{per}^F(T, D, \mu) = -\frac{\hbar^2}{2\pi^2 m} \frac{\partial^2}{\partial D^2} \sum_{p=1}^{\infty} \frac{\rho_1^F(pD)}{p^2}. \quad (23)$$

Before concluding this section, we note that the relationships between the Casimir forces corresponding to various boundary conditions follow directly from Equations (10).

$$F_{Dir}(T, D, \mu) = F_{Neu}(T, D, \mu) = F_{per}(T, 2D, \mu). \quad (24)$$

The above equalities hold both for bosons and fermions. We emphasise that the dependence of the Casimir force on boundary conditions becomes more complicated in the case of non-planar walls (see [5,21-23]).

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An exponential rule governs the long-range behaviour of the one-particle density matrix of a Bose gas in the absence of condensation (0, or c)[1].

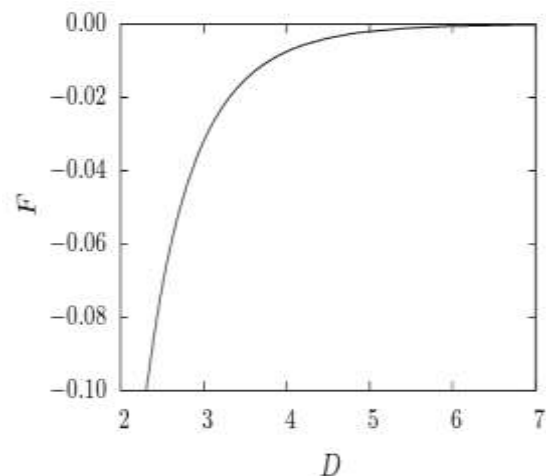
$$\rho_1^B(r) \sim \frac{1}{r\lambda^2} \exp\left[-\frac{2r}{\lambda} \sqrt{\frac{\pi(-\mu)}{k_B T}}\right]. \quad (25)$$

$$\kappa_{per}^B = \frac{\lambda}{2} \sqrt{\frac{k_B T}{\pi(-\mu)}} = \frac{\hbar}{\sqrt{2m(-\mu)}}. \quad (26)$$

Equation (24) allows us to quickly get the intervals that correspond to the Dirichlet and Neumann boundary conditions B.

$$\kappa_{Dir}^B = \kappa_{Neu}^B = \frac{1}{2} \kappa_{per}^B = \frac{\hbar}{2\sqrt{2m(-\mu)}}. \quad (27)$$

According to (26), (27), the force range diverges with a critical exponent $\nu=1/2$ as one approaches the condensate-containing phase (0). An off-diagonal long-range order arises in the two-phase area ($=0$), and the one-particle density matrix $B_1(r)$ approaches for ray nonzero value ($=c$) after a transition.



Casimir force for a Bose gas as a function of separation from walls, shown in Fig. 1. Casimir force is measured in units of $2\hbar^2 m^{-1}$. The units of measure for this distance are.

the power law [1]

$$\rho_1^B(r) - (\rho - \rho_c) \approx \frac{1}{r\lambda^2}. \quad (28)$$

According to Eq. (22) the corresponding decay of the Casimir force for $D \gg \lambda$ is given by

$$F_{per}^B(T, D, 0) = -\frac{\hbar^2}{2\pi^2 m} \frac{\partial^2}{\partial D^2} \left[\sum_{p=1}^{\infty} \frac{1}{p^3 D \lambda^2} \right] = -\frac{2k_B T \zeta(3)}{\pi D^3}, \quad (29)$$

Here, the Riemann zeta function $\zeta(3) \approx 1.202$ is used. It is important to remember that at high distances D the Casimir force $F_{Dir}(T, D, 0)$ is proportional to $(D)^{-3}$, which is much less than the bulk pressure of the ideal Bose gas $p = k_B T \zeta(3) / \lambda^3$ in the two-phase regime. With the help of Eq. (24), we can also quickly determine [8,9].

$$F_{Dir}^B(T, D, 0) = F_{Neu}^B(T, D, \mu) = -\frac{k_B T \zeta(3)}{4\pi D^3}. \quad (30)$$

From Eq. (24), we may infer the behaviour of the correlation function (4). In the zero-phase area, the correlation function $B_2(r)$ is equal to $[B_1(r)]^2$. There is an exponential damping of correlations with respect to the correlation length B in the bulk, such that

$$\xi^B = \kappa_{Dir}^B = \kappa_{Neu}^B = \frac{1}{2} \kappa_{per}^B \quad (31)$$

Casimir forces under Dirichlet and Neumann boundary conditions have a range equal to the correlation length B . An explanation for this surprising agreement may be found in our first-order equation (22). The crucial exponent for the divergence of B and the range of Casimir forces is $1/2$ when $T=0$. Similar conclusions were found in Ref.[15] for the imperfect Bose gas [19,20], which is in a different universality class than the ideal Bose gas [17]. In the absence of a phase transition, the decay durations of correlations and Casimir forces in a Fermi gas diverge for $T=0$. By solving for x in Equation (23), we can determine the precise relationship between the correlation length and the force spectrum. The one-particle density matrix of a perfect Fermi gas is given by the formula [18]

$$\rho_1^F(r) = -\frac{1}{\lambda^3} \sum_{j=1}^{\infty} \frac{(-1)^j}{j^{3/2}} \exp\left[-\frac{\pi r^2}{j \lambda^2}\right]. \quad (32)$$

The following asymptotic formula holds [18] for the characteristics of $F_1(r)$ at low temperatures, which piques our attention.

$$\rho_1^F(r) \Big|_{T \rightarrow 0} = -\frac{1}{\lambda^2 k_F r} \frac{\partial}{\partial r} \left[\frac{\sin(k_F r)}{\sinh(2\pi^2 r / \lambda^2 k_F)} \right] \quad (33)$$

With the Fermi wave vector $k_F = (6\pi^2 \rho)^{1/3}$. The larger behavior involves these exponentially damped oscillations with the characteristic decay length $\lambda^2 / 2k_F$. The connection between the decay times associated with various boundary conditions may be determined using Eqs. (23) and (24).

$$\kappa_{Dir}^F = \kappa_{Neu}^F = \frac{1}{2} \kappa_{per}^F = \frac{\lambda^2 k_F}{4\pi^2}. \quad (34)$$

When $T > 0$, the correlation function $F_2(r) = [F_1(r)]^2$, etc. We infer from (2) that the strength of all correlations decays through damped oscillations proportional to their length.

$$\xi^F = \frac{\lambda^2 k_F}{4\pi^2} = \kappa_{Dir}^F = \kappa_{Neu}^F. \quad (35)$$

For $B = B_{Dir} = B_{Neu}$, we have a full analogue to the Bose gases, and at zero degrees Celsius, Eq. (33) has the form [18]:

$$\rho_1^F(r) \Big|_{T=0} = \frac{\sin(k_F r) - k_F r \cos(k_F r)}{2\pi^2 r^3}. \quad (36)$$

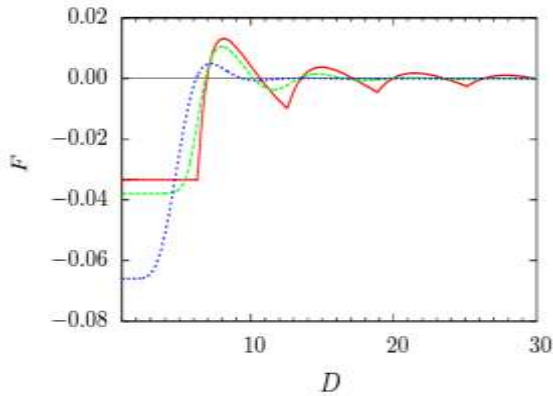
When we plug this formula into (23) we see that the dominating contribution to the Casimir force for D looks like this:

$$F_{per}^F(0, D, \mu_F) = -\frac{\hbar^2 k_F^3}{m\pi^2} \sum_{p=1}^{\infty} \frac{\cos(pk_F D)}{(pD)^2} \quad (37)$$

$$= -\frac{1}{D^2} \left(\frac{\hbar^2 k_F^3}{m} \right) \left[\frac{1}{6} - \left(\frac{k_F D}{2\pi} - \left[\frac{k_F D}{2\pi} \right] \right) + \left(\frac{k_F D}{2\pi} - \left[\frac{k_F D}{2\pi} \right] \right)^2 \right].$$

The Casimir force displays a typical fermionic oscillatory behavior which is superimposed on the power-law decay, see Fig. 2. Keep in mind that the sign of the Casimir force changes as the distance between the walls grows; as the distance grows, the force alternates between being attracting and repulsive, with an endless number of stability points. As the temperature drops, the amplitude of these oscillations grows. At zero temperature, a periodic function is responsible for the oscillations.

$$D \rightarrow \left(\frac{k_F D}{2\pi} - \left[\frac{k_F D}{2\pi} \right] \right)$$



Casimir force for the ideal Fermi gas as a function of wall separation at three temperatures is shown in Fig. 2. A two h2k5 F2m1 measurement of the Casimir force is standard. In this case, we have a distance measured in k1 F units. Temperature is expressed as the metric $t = 2 \cdot 22k2 F m1 T$, which is a dimensionless quantity. Blue solid line represents $t = 0$, red dashed line represents $t = 0.2$, and green dotted line represents $t = 0.5$. (Online colour illustration) Just substituting $2D$ for D in formula (37) yields the Dirichlet and Neumann boundary condition equivalents. The correlation coefficient equals at time $t = 0$ when:

$$\chi^F(r) = -\frac{1}{(2\pi^2)^2} \left[\frac{\sin(k_F r)}{r^3} - \frac{k_F \cos(k_F r)}{r^2} \right]^2 \quad (38)$$

Concluding Comments

In the scenario of ideal quantum gases contained in a slit formed by two infinite parallel walls separated by a distance D , we have examined the relationship between the decay lengths describing the Casimir forces and the bulk correlation lengths. Several boundary conditions (periodic, Dirichlet, and Neumann) were explored for both bosons and fermions. Using a microscopic strategy, we have developed a fundamental formula connecting the Casimir force to the density matrix of a single particle. Its structure, Ems. (22) and (23), is the same for both bosons and fermions (23). The correlation function is directly connected to the one-particle density matrix in the case of ideal quantum gases. It is commonly known that the Casimir force is affected by the boundary conditions chosen, but the bulk correlation function is unaffected. The boundary conditions do not affect the overall structure of the four fundamental formulas, but they do affect which arguments are used for the functions. Both Ems. (22) and (23) represent the impact of boundary conditions on Casimir forces (23). The case of the ideal Bose gas is particularly interesting due to the Bose-Einstein condensing, which happens

when is equal to zero. In the limit where 0 , the Casimir force decays exponentially with a decay length B , resulting to thermodynamic states that correspond to the phase with no condensate. We verified that for a constant temperature $T T_c$, $B \text{ Dir} = B \text{ Neu} = B \text{ Ber}/2 = B ()/2$, where B indicates the correlation length of an ideal Bose gas. Ideal Fermi gas has the same kinds of relationships. A phase transition or critical point is not present, yet the Casimir force and the bulk correlation function exhibit exponentially damped oscillations with increasing amplitude in the limit $T \rightarrow 0$. As the distance between the walls is increased, the Casimir force alternates between being attracting and repulsive, and this results in the aforementioned oscillations. The fermionic bulk correlation length F may be related to the equivalent decay length F . The proportionality to T^{-1} holds at a certain density, and it is seen that these relations are the same as in the case of bosons, where $F \text{ Dir} = F \text{ Neu} = F \text{ Ber}/2 = F T^{-1}$.

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